Two-point correlation function with pion in QCD sum rules

Hungchong Kim[1] [*], Su Houng Lee[2,3][†] and Makoto Oka[1]

[1] *Department of Physics, Tokyo Institute of Technology, Tokyo 152-8551, Japan*
[2] *GSI, Planckstr. 1, D-64291 Darmstadt, Germany*
[3] *Department of Physics, Yonsei University, Seoul 120-749, Korea*

Abstract

Within the framework of the conventional QCD sum rules, we study the pion two-point correlation function, $i \int d^4x e^{iq \cdot x} \langle 0|T J_N(x) \bar{J}_N(0)|\pi(p)\rangle$, beyond the soft-pion limit. We construct sum rules from the three distinct Dirac structures, $i\gamma_5 \not{p}$, $i\gamma_5$, $\gamma_5 \sigma_{\mu\nu} q^\mu p^\nu$ and study the reliability of each sum rule. The sum rule from the third structure is found to be insensitive to the continuum threshold, S_π, and contains relatively small contribution from the undetermined single pole which we denote as b. The sum rule from the $i\gamma_5$ structure is very different even though it contains similar contributions from S_π and b as the ones coming from the $\gamma_5 \sigma_{\mu\nu} q^\mu p^\nu$ structure. On the other hand, the sum rule from the $i\gamma_5 \not{p}$ structure has strong dependence on both S_π and b, which is clearly in constrast with the sum rule for $\gamma_5 \sigma_{\mu\nu} q^\mu p^\nu$. We identify the source of the sensitivity for each of the sum rules by making specific models for higher resonance contributions and discuss the implication.

PACS: 13.75.Gx; 12.38.Lg; 11.55.Hx

Typeset using REVTEX

---

[*]E-mail : hckim@th.phys.titech.ac.jp

[†]AvH fellow.

I. INTRODUCTION

Since first introduced by Shifman, Vainshtein and Zakharov [1], QCD sum rule has been widely used to study the properties of the hadrons [2]. QCD sum rule is a framework which connects a physical parameter to the parameters of QCD. In this framework, a correlation function is introduced in terms of interpolating fields, which are constructed from quark and gluon fields. Then, the correlation function, on the one hand, is calculated by Wilson's operator product expansion (OPE) and, on the other hand, its phenomenological "ansatz" is constructed. A physical quantity of interest is extracted by matching the two descriptions in the deep Euclidean region ($q^2 = -\infty$) via the dispersion relation. The extracted value therefore should be independent of the possible ansatz in order to be phyically meaningful.

The two-point correlation function with pion,

$$\Pi(q, p) = i \int d^4 x e^{iq \cdot x} \langle 0|T[J_N(x)\bar{J}_N(0)]|\pi(p)\rangle , \tag{1}$$

is often used to calculate the pion-nucleon coupling, $g_{\pi N}$, in QCD sum rules [2–4]. Reinders, Rubinstein and Yazaki [2] calculated $g_{\pi N}$ by retaining only the first nonperturbative term in the OPE. Later Shiomi and Hatsuda (SH) [3] improved the calculation by including higher order terms in the OPE. SH considered Eq. (1) and evaluated the OPE in the soft-pion limit ($p_\mu \to 0$).

More recently, Birse and Krippa (BK) [4] pointed out that the use of the soft-pion limit does not constitute an independent sum rule from the nucleon sum rule because in the limit the correlation function is just a chiral rotation of the nucleon correlation function,

$$\Pi(q) = i \int d^4 x e^{iq \cdot x} \langle 0|T[J_N(x)\bar{J}_N(0)]|0\rangle . \tag{2}$$

Therefore, BK considered the sum rule beyond the soft-pion limit. However, as we will discuss below, there seems to be mistakes in their calculation which can invalidate their conclusions. Thus, it is important to re-do their calculation.

In a recent letter [6], we have pointed out that the previous calculations of the pion-nucleon coupling using Eq. (1) have dependence on how one models the phenomenological side; either using the pseudoscalar (PS) or the pseudovector (PV) coupling scheme. The two coupling schemes are equivalent when the participating nucleons are on-shell but they are not usually when the nucleons are off-shell. Since, in QCD sum rules, on-shell properties of a particle are extracted from the far off-shell point, the extracted $g_{\pi N}$ therefore could be coupling-scheme dependent. Going beyond the soft-pion limit is found to be also natural in obtaining $g_{\pi N}$ independent of the PS and PV coupling schemes. In fact, we have proposed that, beyond the soft-pion limit, there are three distinct Dirac structures, (1) $i\gamma_5 \not{p}$, (2) $i\gamma_5$, (3) $\gamma_5\sigma_{\mu\nu}q^\mu p^\nu$, each of which can in principle be used to calculate $g_{\pi N}$. The third structure was found to have the common double pole structure in the phenomenological side, independent of the PS and PV coupling schemes. By studying this structure, we obtained the coupling close to its empirical value and relatively stable against the uncertainties from QCD parameters. Then we ask, can we get similar stable results from the sum rules constructed from the other Dirac structures ? If not, what are the reasons for the differences ? In this work, we will try to answer these questions by studying these three sum rules and investigating the reliability of each sum rule.

QCD sum rules could depend on a specific Dirac structure considered. This aspect was suggested by Jin and Tang [7] in their study of baryon sum rules. They found that the chiral odd sum rule is more reliable due to the partial cancellation of the positive and negative-parity excited baryons in the continuum. Similarly here we note that the structure (1) has different chirality from the other two. Therefore it will be interesting to look into these sum rules more closely and see if similar cancellation occurs for certain sum rules.

The paper is organized as follows. In Section II, we construct three sum rules from the three different Dirac structures. The spectral density for the phenomenological side is constructed from the double pole, the unknown single pole and the continuum modeled by a step function. We motivate this phenomenological spectral density in Section III by using some effective Lagrangians for the transitions, $N \to N^*$ and $N^* \to N^*$. In Section IV, we analyze each sum rule and try to understand the differences from the formalism constructed in Section III. A summary is given in Section V.

II. QCD SUM RULES FOR THE TWO-POINT CORRELATION FUNCTION

In this section, we formulate three different sum rules for the two-point correlation function with pion beyond the soft-pion limit. For technical simplicity, we consider the correlation function with charged pion,

$$\Pi(q, p) = i \int d^4 x e^{iq \cdot x} \langle 0|T[J_p(x)\bar{J}_n(0)]|\pi^+(p)\rangle \ . \tag{3}$$

Here J_p is the proton interpolating field suggested by Ioffe [8],

$$J_p = \epsilon_{abc}[u_a^T C \gamma_\mu u_b]\gamma_5 \gamma^\mu d_c \tag{4}$$

and the neutron interpolating field J_n is obtained by replacing $(u, d) \to (d, u)$. In the OPE, we only keep the diquark component of the pion wave function and use the vacuum saturation hypothesis to factor out higher dimensional operators in terms of the pion wave function and the vacuum expectation value.

The calculation of the correlator, Eq. (3), in the coordinate space contains the following diquark component of the pion wave function,

$$D_{aa'}^{\alpha\beta} \equiv \langle 0|u_a^\alpha(x)\bar{d}_{a'}^\beta(0)|\pi^+(p)\rangle \ . \tag{5}$$

Here, α and β are Dirac indices, a and a' are color indices. The other quarks are contracted to form quark propagators. This diquark component can be written in terms of three Dirac structures,

$$D_{aa'}^{\alpha\beta} = \frac{\delta_{aa'}}{12}(\gamma^\mu \gamma_5)^{\alpha\beta}\langle 0|\bar{d}(0)\gamma_\mu\gamma_5 u(x)|\pi^+(p)\rangle \ + \frac{\delta_{aa'}}{12}(i\gamma_5)^{\alpha\beta}\langle 0|\bar{d}(0)i\gamma_5 u(x)|\pi^+(p)\rangle$$

$$-\frac{\delta_{aa'}}{24}(\gamma_5\sigma^{\mu\nu})^{\alpha\beta}\langle 0|\bar{d}(0)\gamma_5\sigma_{\mu\nu}u(x)|\pi^+(p)\rangle \ . \tag{6}$$

Each matrix element associated with each Dirac structure can be written in terms of pion wave function whose first few moments are relatively well known [9]. We will come back to the second matrix element later. For the other two elements, we need only the normalization

of the pion wave functions since we are doing the calculation up to the first order in p_μ. In fact, to leading order in the pion momentum, the first and third matrix elements are given as [9],

$$\langle 0|\bar{d}(0)\gamma_\mu\gamma_5 u(x)|\pi^+(p)\rangle = i\sqrt{2}f_\pi p_\mu + \text{twist 4 term} , \tag{7}$$

$$\langle 0|\bar{d}(0)\gamma_5\sigma_{\mu\nu} u(x)|\pi^+(p)\rangle = i\sqrt{2}(p_\mu x_\nu - p_\nu x_\mu)\frac{f_\pi m_\pi^2}{6(m_u + m_d)} . \tag{8}$$

Here we have suppressed terms higher order in pion momentum. The factor $\sqrt{2}$ is just an isospin factor. The twist 4 term in Eq. (7) comes from the second derivative term in the short distant expansion of the LHS. Note that in Eq. (8) the factor $f_\pi m_\pi^2/(m_u + m_d)$ can be written as $-\langle\bar{q}q\rangle/f_\pi$ by making use of Gell-Mann−Oakes−Renner relation. Although the operator looks gauge dependent, it is understood that the fixed point gauge is used throughout and the final result is gauge independent. It is then interesting to note that the LHS of Eq. (8) can also be expanded in x such that the matrix element that contributes is effectively one with higher dimension,

$$\langle 0|\bar{d}(0)\gamma_5\sigma_{\mu\nu} D_\alpha u(0)|\pi^+(p)\rangle = i\sqrt{2}(p_\mu g_{\alpha\nu} - p_\nu g_{\alpha\mu})\frac{f_\pi m_\pi^2}{6(m_u + m_d)} . \tag{9}$$

It is now straightforward to calculate the OPE. For the $i\gamma_5 \not{p}$ structure, we obtain

$$\sqrt{2}f_\pi \left[\frac{q^2\ln(-q^2)}{2\pi^2} + \frac{\delta^2\ln(-q^2)}{2\pi^2} + \frac{\left\langle\frac{\alpha_s}{\pi}\mathcal{G}^2\right\rangle}{12q^2} + \frac{2\langle\bar{q}q\rangle^2}{9f_\pi^2 q^2}\right] . \tag{10}$$

The first three terms are obtained by taking the second term in Eq. (6)[1], while the fourth term is obtained by taking the third term in Eq.(6) and replacing one quark propagator with the quark condensate. The fourth term was not taken into account in the sum rule studied by BK [4] but its magnitude is about 4 times larger than the third term. So there is no reason to neglect the fourth term while keeping the third term. The second term comes from the twist-4 element of pion wave function. According to Novikov $et.$ al [11], $\delta^2 \sim 0.2$ GeV^2.

The phenomenological side for the $i\gamma_5 \not{p}$ structure obtained by using the pseudoscalar Lagrangian takes the form,

$$-\frac{\sqrt{2}g_{\pi N}\lambda_N^2 m}{(q^2 - m^2 + i\epsilon)[(q-p)^2 - m^2 + i\epsilon]} + \cdots . \tag{11}$$

The dots include contributions from the continuum as well as from the unknown single pole terms . The latter consists of the single pole coming from $N \to N^*$ transition [10]. When the pseudovector Lagrangian is used, there is an additional single pole coming from $N \to N$ transition [6]. These single poles are not suppressed by the Borel transformation.

---

[1]Note that the second term in Eq. (10) has slightly different coefficient from BK [4]. Ref. [4] has the factor 5/9 instead of our factor 1/2. The difference however is small.

Therefore, interpretation of the unknown single pole and possibly the continuum contain some ambiguity due to the coupling scheme adopted.

In principle, $g_{\pi N}$ has p^2 dependence as it contains the pion form factor. As one pion momentum is taken out by the Dirac structure, $i\gamma_5 \not{p}$, we take $p_\mu = 0$ in the rest of the correlator as we did in the OPE side. Then the p^2 dependence of $g_{\pi N}$ can be neglected. Furthermore, after taking out the factor, $i\gamma_5 \not{p}$, the rest of the correlator is a function of one variable, q^2, and therefore the single dispersion relation in q^2 can be invoked in constructing the sum rule. Anyhow, the spectral density can be written as

$$\rho^{phen}(s) = -\sqrt{2}g_{\pi N}\lambda_N^2 m \frac{d}{ds}\delta(s-m^2) + A'\,\delta(s-m^2) + \rho^{ope}(s)\,\theta(s-S_\pi)\ . \tag{12}$$

Here the second term comes from the single pole terms whose coefficients are not known. The continuum contribution is parameterized by a step function which starts from the threshold, S_π. The coefficient of the step function, $\rho^{ope}(s)$, is determined by the duality of QCD. This is basically the imaginary part of Eq. (10) but, because of the continuum threshold, only the first two terms in Eq. (10) contribute to the coefficient.

The parameterization of the continuum with a step function is usually adopted in the baryon mass sum rules. This is because each higher resonance has a single pole structure with a finite width. Spectral density obtained by adding up all those single poles can be effectively represented by a step function starting from a threshold. But in our case of the correlation function with pion, this parameterization for the continuum could be questionable. Therefore, it will be useful to construct the spectral density explicitly for higher resonances by employing some effective models for N^* and see if the parameterization does make sense. This will be done in the next section. This will eventually help us to understand how each sum rule based on a different Dirac structure leads to different results.

To construct QCD sum rule for the $i\gamma_5 \not{p}$ structure, we integrate $\rho^{ope}(s)$ and $\rho^{phen}(s)$ with the Borel weighting factor e^{-s/M^2} and match both sides. More specifically, the sum rule equation after the Borel transformation is given by

$$\int_0^\infty ds e^{-s/M^2}[\rho^{ope}(s) - \rho^{phen}(s)] = 0\ . \tag{13}$$

Using $\rho^{ope}(s)$ obtained from Eq. (10) and $\rho^{phen}(s)$ in Eq. (12), we obtain

$$
\begin{aligned}
&g_{\pi N}\lambda_N^2(1 + AM^2)\\
&= \frac{f_\pi}{m}e^{m^2/M^2}\left[\frac{E_1(x_\pi)}{2\pi^2}M^6 + \frac{E_0(x_\pi)}{2\pi^2}M^4\delta^2 + M^2\left(\frac{1}{12}\left\langle\frac{\alpha_s}{\pi}\mathcal{G}^2\right\rangle + \frac{2\langle\bar{q}q\rangle^2}{9f_\pi^2}\right)\right]\ .
\end{aligned}
\tag{14}
$$

Here A denotes the unknown single pole contribution, which should be determined by the best fitting method. Also $x_\pi = S_\pi/M^2$ and $E_n(x) = 1 - (1 + x + \cdots + x^n/n!)\,e^{-x}$. This expression is crucially different from the corresponding expression in Ref. [4] where the first, second and third terms contain the factors, $E_2(x_\pi)$, $E_1(x_\pi)$ and $E_0(x_\pi)$ respectively. Even though we do not understand how such factors can be obtained, we nevertheless reproduce their figure by using their formula in Ref. [4] and it is shown in Fig. 1 (a)[2]. But if Eq. (14) is

---

[2]In plotting Figs. 1, we did not include the last term involving $\langle\bar{q}q\rangle^2$ in Eq. (14) as this term is new in our calculation.

used instead, we get Fig. 1 (b) using the same parameter set used in Ref. [4]. The variation scale of $g_{\pi N}$ in this figure is clearly different from the one in Fig. 1 (a). Note that some of their parameters are quite different from ours used in our analysis later part of this work. For example, $\delta^2 = 0.35$ GeV^2 is used in Ref. [4], which is quite larger than our value of 0.2 GeV^2.

QCD sum rule for the $\gamma_5\sigma_{\mu\nu}q^\mu p^\nu$ structure can be constructed similarly. We have constructed the sum rule for this structure in Ref. [6] so here we simply write down the resulting expression,

$$g_{\pi N}\lambda_N^2(1+BM^2) = -\frac{\langle\bar{q}q\rangle}{f_\pi}e^{m^2/M^2}\left[\frac{M^4E_0(x_\pi)}{12\pi^2} + \frac{4}{3}f_\pi^2M^2 + \left\langle\frac{\alpha_s}{\pi}\mathcal{G}^2\right\rangle\frac{1}{216} - \frac{m_0^2f_\pi^2}{6}\right] . \quad (15)$$

Here B denotes the contribution from the unknown single pole term. Note that, since one power of the pion momentum is taken out by the factor, $\gamma_5\sigma_{\mu\nu}q^\mu p^\nu$, we take the limit $p_\mu = 0$ in the rest of the correlator as we did in the $i\gamma_5 \not{p}$ case. In obtaining the first and third terms in RHS, we have used Eq. (8) while the second is obtained by taking the first term in Eq. (6) for the matrix element $D_{aa'}^{\alpha\beta}$ and replacing one propagator with the quark condensate. The fourth term is also obtained by taking the first term in Eq. (6) but in this case other quarks are used to form the dimension five mixed condensate, $\langle\bar{q}g_s\sigma\cdot\mathcal{G}q\rangle$, which is usually parameterized in terms of the quark condensate, $m_0^2\langle\bar{q}q\rangle$. We take $m_0^2 \sim 0.8$ GeV^2 as obtained from QCD sum rule calculation [12].

Now we construct QCD sum rule for the $i\gamma_5$ structure. Constructing it beyond the soft-pion limit is more complicated as the correlator in phenomenological side has definite dependence on the coupling schemes. To see this, we expand the correlator for this structure in p_μ and write

$$\Pi_0(q^2) + p\cdot q\Pi_1(q^2) + p^2\Pi_2(q^2) + \cdots . \quad (16)$$

Since p_μ is an external momentum, the correlation function at each order of p_μ can be used to construct an independent sum rule. Within the PS coupling scheme, the phenomenological correlator up to p^2 order is

$$\sqrt{2}g_{\pi N}\lambda_N^2\left[-\frac{1}{q^2-m^2} - \frac{p\cdot q}{(q^2-m^2)^2} + \frac{p^2}{(q^2-m^2)^2}\right] - \frac{\sqrt{2}\lambda_N^2p^2}{q^2-m^2}\frac{dg_{\pi N}}{dp^2}(p^2=0)\cdots . \quad (17)$$

The dots here represent not only the contribution from higher resonances but also terms higher than p^2. The last term is related to the slope of the pion form factor at $p^2 = 0$. Even though this can be absorbed into the unknown single pole term such as A or B above, we specify it here since this possibility is new. This correlator can be compared with the corresponding expression in the PV coupling scheme,

$$\sqrt{2}g_{\pi N}\lambda_N^2\frac{p^2/2}{(q^2-m^2)^2} + \cdots . \quad (18)$$

Note here that there are no terms corresponding to Π_0 and Π_1. No such terms can be constructed from $N \to N^*$ or $N^* \to N^*$ transitions within the PV scheme.

The single pole in Eq.(17) survives in the soft-pion limit, which has been used by SH [3] for their sum rule calculation of $g_{\pi N}$. However, if the phenomenological correlator in PV

scheme is used, such sum rule cannot be constructed. Thus, going beyond the soft-pion limit seems to be natural for the independent determination of the coupling. However, similarly for Π_0 case, a sum rule can not be constructed for Π_1. For Π_2, a sum rule can be constructed either in the PS or PV coupling scheme, but the residue of the double pole in Eq. (18) is a factor of two smaller than the corresponding term in Eq. (17). So the coupling-scheme independence can not be achieved in any of these sum rules. This is true for even higher orders of p_μ.

A sum rule, independent of the coupling schemes, can be constructed by imposing the kinematical condition,

$$p^2 = 2p \cdot q . \tag{19}$$

With this condition, the two double pole terms in Eq. (17) can be combined to yield the same expression as in Eq. (18), thus providing a sum rule independent of the coupling schemes. This condition comes from the on-shell conditions for the participating nucleons, $q^2 = m^2$ and $(q - p)^2 = m^2$, at which the physical πNN coupling should be defined.

The sum rule constructed with the kinematical condition, Eq. (19), is equivalent to consider $\Pi_1(q^2)/2 + \Pi_2(q^2)$. This sum rule seems fine in the PS coupling scheme as there are nonzero terms corresponding to Π_1 and Π_2. In the OPE, the diquark component contributing to $i\gamma_5$ structure is the second element of Eq. (6) which can be written in terms of twist-3 pion wave function as [9]

$$\langle 0|\bar{d}(0)i\gamma_5 u(x)|\pi^+(p)\rangle = \frac{\sqrt{2}f_\pi m_\pi^2}{m_u + m_d} \int_0^1 du e^{-iup \cdot x} \varphi_p(u) . \tag{20}$$

The terms linear and quadratic in p_μ in the RHS constitute the OPE correlator for Π_1 and Π_2. Therefore, within the PS scheme, Π_1 and Π_2 are well defined in both sides.

Situation becomes subtle when the PV coupling scheme is employed. Before the condition of Eq. (19) is imposed, a sum rule can be constructed only for Π_2 as there is no Π_1 part in the phenomenological part. But after the condition, the phenomenological side has only Π_2^{phen} which should be matched with $\Pi_1^{ope}/2 + \Pi_2^{ope}$. This seems a little awkward. Nevertheless, to achieve the independence of the coupling schemes, we construct a QCD sum rule for $i\gamma_5$ within the kinematical condition, Eq. (19). To be consistent with the expansion in the phenomenological side, we take the terms up to the order p^2 in the expansion of Eq. (20). Using the parameterization for $\varphi_p(u)$ given in Ref. [9], we obtained up to p^2,

$$\langle 0|\bar{d}(0)i\gamma_5 u(x)|\pi^+(p)\rangle = \frac{\sqrt{2}f_\pi m_\pi^2}{m_u + m_d} \left(1 - i\frac{1}{2}p \cdot x - \frac{0.343}{2}(p \cdot x)^2\right) . \tag{21}$$

A different parameterization given in Ref. [9] changes the numerical factors very slightly .

Using the diquark component of Eq. (21), the OPE side for $\Pi_1/2 + \Pi_2$ is calculated straightforwardly. By matching with its phenomenological counterpart and taking the Borel transformation, we get

$$g_{\pi N}\lambda_N^2(1 + CM^2) = \frac{\langle \bar{q}q \rangle}{f_\pi}e^{m^2/M^2}\left[\frac{0.0785E_0(x_\pi)}{\pi^2}M^4 - 0.314 \times \frac{1}{24}\left\langle \frac{\alpha_s}{\pi}\mathcal{G}^2\right\rangle\right] . \tag{22}$$

Here C again denotes the unknown single pole term which is not suppressed by the Borel transformation. This sum rule is different from the other two sum rules as its first term in

the OPE is negative. Each term contains very small numerical factors due to the cancellation between Π_1^{ope} and Π_2^{ope}.

Up to now, we have presented three different sum rules from Eq. (1). All these sum rules, in principle, can be used to determine the pion-nucleon coupling constant, $g_{\pi N}$. We will discuss the reliability of each sum rule below. An alternative approach is to consider the nucleon correlation function in an external axial field as done in Ref [5]. The nucleon axial charge, g_A, calculated in Ref. [5], agree well with experiment. Subsequently, by using the Goldberger-Treiman relation, $g_{\pi N}$ can be also well determined. In the approach by Ref. [5], sum rules for $g_A - 1$ is obtained by replacing some part of the OPE with the nucleon mass sum rule. The connection between the sum rules using Eq. (1) and the ones in Ref. [5] is not clear at this moment. An important observation made in Ref. [5] is to note that some (dominant) terms of the OPE correspond to a sum rule with $g_A = 1$. This observation allows the construction of the sum rules for $g_A - 1$. In our sum rules, this kind of observation is not possible. Also the OPE expression from Eq. (1) is not simply related to the sum rules with the external field. Therefore, Eq. (1) seems to provide independent sum rules from the ones in Ref. [5]. In future, however, further study is necessary to clarify the connection between these two sets of independent sum rules as it might provide important aspects for the nonperturbative nature of hadrons.

III. CONSTRUCTION OF THE UNKNOWN SINGLE POLE AND THE CONTINUUM

In this section, we construct the unknown single pole term and the continuum by using effective models for the higher resonances. This will provide a better understanding of the parameterization for the continuum in Eq. (12) and give further insights for the unknown single pole term. Later, this construction will help us to understand the differences between each sum rule based on a different Dirac structure.

There are two possible sources for the unknown single pole term and the continuum. One is from the transition, $N \to N^*$ and the other is from the transition, $N^* \to N^*$. Of course, as we pointed out in Ref. [6], there could be additional single pole of nucleon coming from $N \to N$, which however, in the first order of the pion momentum, appears only in the sum rule for the $i\gamma_5 \not{p}$ structure within the PV coupling scheme. First we avoid such possibility by constructing effective models within the PS coupling scheme. Later we will discuss the case with the PV coupling scheme. Moreover, we will discuss only the two Dirac structures, $i\gamma_5 \not{p}$ and $\gamma_5 \sigma_{\mu\nu} q^\mu p^\nu$. The correlator for the $i\gamma_5$ structure with the kinematical condition of Eq. (19) takes almost the same form as the one for the $\gamma_5 \sigma_{\mu\nu} q^\mu p^\nu$ structure. A slight difference is the appearance of terms containing the derivative of pion form factor as indicated in Eq. (17). Note that this difference is only specific to the PS coupling scheme. As the form factor is a smooth function of p^2 around $p^2 = 0$, this difference is not expected to be crucial.

Within the PS coupling scheme, $N \to N^*$ contributions to the correlator, Eq. (3), can be constructed by using the effective Lagrangians for the positive (ψ_+) and negative (ψ_-) parity resonances,

$$g_{\pi N N_+} \bar{\psi} i\gamma_5 \boldsymbol{\tau} \cdot \boldsymbol{\pi} \psi_+ + g_{\pi N N_+} \bar{\psi}_+ i\gamma_5 \boldsymbol{\tau} \cdot \boldsymbol{\pi} \psi \ ,$$

$$g_{\pi N N_-} \bar{\psi} i \boldsymbol{\tau} \cdot \boldsymbol{\pi} \psi_- - g_{\pi N N_-} \bar{\psi}_- i \boldsymbol{\tau} \cdot \boldsymbol{\pi} \psi \ . \tag{23}$$

The nucleon field is denoted by ψ here. These terms contribute to the correlator because the nucleon interpolating field can couple to the positive and negative parity resonances via,

$$\langle 0 | J_N | N_+(k, s) \rangle = \lambda_+ U(k, s) \ ; \quad \langle 0 | J_N | N_-(k, s) \rangle = \lambda_- \gamma_5 U(k, s) \ , \tag{24}$$

where $U(k, s)$ denotes the baryon Dirac spinor and $\lambda_\pm$ indicates the coupling strength of the interpolating field to each resonance with specified parity.

The $\gamma_5 \sigma_{\mu\nu} q^\mu p^\nu$ structure of the correlator takes the form,

$$\frac{2 \lambda_N \lambda_- g_{\pi N N_-}}{(q^2 - m^2)(q^2 - m_-^2)} + \frac{2 \lambda_N \lambda_+ g_{\pi N N_+}}{(q^2 - m^2)(q^2 - m_+^2)} \ , \tag{25}$$

which can be compared with the $i \gamma_5 \not{p}$ structure

$$\frac{2 \lambda_N \lambda_- g_{\pi N N_-}(m_- - m)}{(q^2 - m^2)(q^2 - m_-^2)} - \frac{2 \lambda_N \lambda_+ g_{\pi N N_+}(m_+ + m)}{(q^2 - m^2)(q^2 - m_+^2)} \ . \tag{26}$$

By separating as

$$\frac{1}{(q^2 - m^2)(q^2 - m_\pm^2)} \rightarrow - \frac{1}{m_\pm^2 - m^2} \left[\frac{1}{q^2 - m^2} - \frac{1}{q^2 - m_\pm^2} \right] \tag{27}$$

we can see that the transitions, $N \rightarrow N^*$, involve the two single poles, one with the nucleon pole and the other with the resonance pole. The former constitutes the unknown single pole as it involves the undetermined parameters, $\lambda_\pm$ and $g_{\pi N N_\pm}$. In the latter, the finite width of the resonances can be incorporated by replacing $m_\pm \rightarrow m_\pm - i \Gamma_\pm/2$ in the denominator. Then when it is combined with other such single poles from higher resonances, it produces the spectral density which can be parameterized by a step function as written in Eq. (12). This also implies that the continuum threshold, S_π, does not need to be different from the one appearing in the usual nucleon sum rule.

It is now easy to obtain the spectral density for the two Dirac structures by incorporating the decay width of the resonances. For the $i \gamma_5 \not{p}$ structure, we have

$$\rho_S(s) = 2 \left(- \frac{\lambda_+ g_{\pi N N_+}}{m_+ - m} + \frac{\lambda_- g_{\pi N N_-}}{m_- + m} \right) \lambda_N \delta(s - m^2)$$
$$+ \frac{2 \lambda_N \lambda_+ g_{\pi N N_+}}{m_+ - m} G(s, m_+) - \frac{2 \lambda_N \lambda_- g_{\pi N N_-}}{m_- + m} G(s, m_-) \tag{28}$$

where

$$G(s, m_\pm) = \frac{1}{\pi} \frac{m_\pm \Gamma_\pm}{(s - m_\pm^2)^2 + m_\pm^2 \Gamma_\pm^2} \ . \tag{29}$$

Note that the contribution from the positive-parity resonance is enhanced by the factor $1/(m_+ - m)$ while the one from the negative-parity resonance is suppressed by the factor $1/(m_- + m)$. Similarly for the $\gamma_5 \sigma_{\mu\nu} q^\mu p^\nu$ structure, we obtain

9

$$\rho_S(s) = 2 \left(\frac{\lambda_+ g_{\pi N N_+}}{m_+^2 - m^2} + \frac{\lambda_- g_{\pi N N_-}}{m_-^2 - m^2} \right) \lambda_N \delta(s - m^2)$$
$$- \frac{2\lambda_N \lambda_+ g_{\pi N N_+}}{m_+^2 - m^2} G(s, m_+) - \frac{2\lambda_N \lambda_- g_{\pi N N_-}}{m_-^2 - m^2} G(s, m_-) . \tag{30}$$

Note that the superficial relative sign between the positive- and negative-parity resonances are opposite to that in Eq. (28). It means, depending on the relative sign between $\lambda_+ g_{\pi N N_+}$ and $\lambda_- g_{\pi N N_-}$, the two contributions add up in one case or cancel each other in the other case. In other words, we can say something about the coefficients of $\delta(s - m^2)$ and $G(s, m_\pm)$, by studying the sensitivity of the sum rules to the continuum or to the single pole.

Additional contribution to the continuum may come from $N^* \to N^*$ transitions. For the off-diagonal transitions between two parities, $N_+ \to N_-$ and $N_- \to N_+$, we use the effective Lagrangians,

$$g_{\pi N_+ N_-} \bar{\psi}_+ i \boldsymbol{\tau} \cdot \boldsymbol{\pi} \psi_- - g_{\pi N_+ N_-} \bar{\psi}_- i \boldsymbol{\tau} \cdot \boldsymbol{\pi} \psi_+ , \tag{31}$$

to construct the correlator. These off-diagonal transitions lead to the spectral density of

$$\rho_{OD}(s) \propto \lambda_- \lambda_+ [G(s, m_+) - G(s, m_-)] , \tag{32}$$

which is therefore suppressed by the cancellation between the two parity resonances.

For the diagonal transitions, $N_+ \to N_+$ and $N_- \to N_-$, we use the effective Lagrangians,

$$g_{\pi N_+ N_+} \bar{\psi}_+ i \gamma_5 \boldsymbol{\tau} \cdot \boldsymbol{\pi} \psi_+ ; \quad g_{\pi N_- N_-} \bar{\psi}_- i \gamma_5 \boldsymbol{\tau} \cdot \boldsymbol{\pi} \psi_- . \tag{33}$$

These diagonal transitions produce only the double pole for the correlator, $1/(q^2 - m_\pm^2 + i m_\pm \Gamma_\pm)^2$, which is then translated into the spectral density,

$$\rho_D(s) \sim \begin{cases} -m_\pm g_{\pi N_\pm N_\pm} \lambda_\pm^2 \frac{d}{ds} G(s, m_\pm) & \text{for } i\gamma_5 \not{p} \\ \pm g_{\pi N_\pm N_\pm} \lambda_\pm^2 \frac{d}{ds} G(s, m_\pm) & \text{for } \gamma_5 \sigma_{\mu\nu} q^\mu p^\nu . \end{cases} \tag{34}$$

First note that, because of the derivative, each spectral density has a node at $s = m_\pm^2$, positive below the resonance and negative above the resonance. Then under the integration over s, the spectral density from the double pole is partially canceled, leaving attenuated contribution coming from the s dependent Borel weight. Indeed, one can numerically check that, for the Roper resonance, $\int ds e^{-s/M^2} G(s, m_+)$ is always larger than $\int ds e^{-s/M^2} dG(s, m_+)/ds$ for $M^2 \geq 0.7$ GeV^2 and the cancellation is more effective as M^2 increases. In general, the continuum contributes more to a sum rule for larger M^2. Hence the double pole is more suppressed than the single pole in the region where the continuum is large. Further suppression of the double pole continuum can be observed, for example, by comparing the first equation of Eq. (34) with Eq. (28). Even if one assumes [3] that $g_{\pi N N_+} \lambda_N \lambda_+ \sim g_{\pi N_+ N_+} \lambda_+^2$,

---

[3] The nucleon interpolating field, J_N, is constructed such that it couples strongly to the nucleon but weakly to excited states. Therefore, λ_+ is expected to be smaller than λ_N. This assumption, therefore, may be regarded as assuming strong coupling to the excited baryon.

then Eq. (28) has the enhancing factor of $1/(m_+ - m)$ while the first equation in Eq. (34) contains only m_+. Thus, the double pole contribution is much suppressed than the single pole, which can be checked also from numerical calculations. The similar suppression can be expected for the second equation in Eq. (34). Therefore, we expect that the continuum mainly comes from the single pole of $1/(q^2 - m_\pm^2 + im_\pm\Gamma_\pm)$ which is generated only from the $N \to N^*$ transitions. This will justify the "step-like" parameterization of the continuum as given in Eq. (12).

Now we discuss the case with the PV coupling scheme. We use the following Lagrangians

$$\frac{g_{\pi N_+ N_+}}{2m_+} \bar{\psi}_+ \gamma_5 \gamma_\mu \boldsymbol{\tau} \cdot \partial^\mu \boldsymbol{\pi} \psi_+ \; ; \quad \frac{g_{\pi N_- N_-}}{2m_-} \bar{\psi}_- \gamma_5 \gamma_\mu \boldsymbol{\tau} \cdot \partial^\mu \boldsymbol{\pi} \psi_- \; ,$$

$$\frac{g_{\pi N N_+}}{m + m_+} \bar{\psi}_+ \gamma_5 \gamma_\mu \boldsymbol{\tau} \cdot \partial^\mu \boldsymbol{\pi} \psi + (H.C.) \; ,$$

$$\frac{g_{\pi N N_-}}{m_- - m} \bar{\psi}_- \gamma_5 \gamma_\mu \boldsymbol{\tau} \cdot \partial^\mu \boldsymbol{\pi} \psi + (H.C.) \; ,$$

$$\frac{g_{\pi N_+ N_-}}{m_- - m_+} \bar{\psi}_- \gamma_5 \gamma_\mu \boldsymbol{\tau} \cdot \partial^\mu \boldsymbol{\pi} \psi_+ + (H.C.) \; . \tag{35}$$

These effective Lagrangians in the PV scheme are constructed such that the action is the same as the PS case when the resonances are on-shell . In this case, complications arise from the possible single pole term coming from $N \to N$ [6] contribution which was absent in the PS scheme. This also means that there could be additional single poles coming from $N \to N^*$ and $N^* \to N^*$ transitions. Note, this kind of complication arises only in the $i\gamma_5 \not{p}$ case. That is, for the $\gamma_5 \sigma_{\mu\nu} q^\mu p^\nu$ case, we have the same spectral density as given in Eq. (30).

As we mentioned above, because the double pole type contribution, $1/(q^2 - m_\pm^2 + im_\pm\Gamma_\pm)^2$, to the continuum is suppressed, only single poles are important in constructing the spectral density for the unknown single pole and the "step-like" continuum. To construct the single poles, we consider all possibilities, $N \to N$, $N \to N^*$ and $N^* \to N^*$. The coefficient of $\lambda_N \delta(s - m^2)$, namely the unknown single pole term for the $i\gamma_5 \not{p}$ structure, can be collected from $N \to N$ and $N \to N^*$ transitions,

$$\frac{g_{\pi N} \lambda_N}{2m} - 2m \left(\frac{\lambda_+ g_{\pi N N_+}}{m_+^2 - m^2} + \frac{\lambda_- g_{\pi N N_-}}{m_-^2 - m^2} \right) \; . \tag{36}$$

Compared with the corresponding term in Eq. (30), this term is differed by the first term associated with $N \to N$. The second and third terms are the same except for the overall factor, $-2m$.

Also the continuum contributions are collected from the terms containing $1/(q^2 - m_\pm^2)$ in the correlator. We thus obtain the spectral density for the continuum,

$$\left(g_{\pi N_+ N_+} \frac{\lambda_+^2}{2m_+} + g_{\pi N N_+} \frac{2m_+ \lambda_N \lambda_+}{m_+^2 - m^2} - g_{\pi N_+ N_-} \frac{2m_+ \lambda_+ \lambda_-}{m_-^2 - m_+^2} \right) G(s, m_+)$$

$$+ \left(g_{\pi N_- N_-} \frac{\lambda_-^2}{2m_-} - g_{\pi N N_-} \frac{2m_- \lambda_N \lambda_-}{m_-^2 - m^2} - g_{\pi N_+ N_-} \frac{2m_- \lambda_+ \lambda_-}{m_-^2 - m_+^2} \right) G(s, m_-) \; . \tag{37}$$

IV. RELIABILITY OF QCD SUM RULES AND POSSIBLE INTERPRETATION

In section II, we have constructed three sum rules, each for the $i\gamma_5 \not{p}$, the $i\gamma_5$ and the $\gamma_5\sigma_{\mu\nu}q^\mu p^\nu$ structures beyond the soft-pion limit. Ideally, all three sum rules should yield the same result for $g_{\pi N}$. In reality, each sum rule could have uncertainties due to the truncation in the OPE side or large contributions from the continuum. Therefore, depending on Dirac structures, there could be large or small uncertainties in the determination of the physical parameter. This can be checked by looking into the Borel curves and seeing whether or not they are stable functions of the Borel mass. In the QCD sum rules for baryon masses, the ratio of two different sum rules is usually taken in extracting a physical mass without explicitly checking the stability of each sum rule. As pointed out by Jin and Tang [7], this could be dangerous. In this section, we will demonstrate this issue further by considering three sum rules provided in section II.

In Eqs. (14), (22) and (15), LHS can be written in the form, $c + bM^2$. The parameter c denotes the same quantity, *i.e.* $g_{\pi N}\lambda_N^2$, but b could be different in each sum rule. We can determine c and b by fitting RHS by a straight line within the appropriately chosen Borel window. Usually, the maximum Borel mass is determined by restricting the continuum contribution to be less than, say, $30 \sim 40$ % of the first term of the OPE and the minimum Borel mass is chosen by restricting the highest dimensional term of the OPE to be less than, say $10 \sim 20$ % of the total OPE. These criteria lead to the Borel window centered around the Borel mass $M^2 \sim 1$ GeV^2. Further notice that c determined in this way does not depend on the PS and PV coupling schemes while the interpretation of b could be scheme-dependent.

In the analysis below, we use the following standard values for the QCD parameters,

$$\langle\bar{q}q\rangle = -(0.23 \text{ GeV})^3 \;;\quad \left\langle\frac{\alpha_s}{\pi}\mathcal{G}^2\right\rangle = (0.33 \text{ GeV})^4 \;,$$
$$\delta^2 = 0.2 \text{ GeV}^2 \;;\quad m_0^2 = 0.8 \text{ GeV}^2 \;. \tag{38}$$

Uncertainties in these parameters do not significantly change our discussion below. For the nucleon mass m and the pion decay constant f_π, we use their physical values, $m = 0.94$ GeV and $f_\pi = 0.093$ GeV.

In Figure 2 (a), we plot the Borel curves obtained from Eqs.(14), (22) and (15). The thick solid line is from Eq.(15), the thick dot-dashed line from Eq.(22) and the thick dashed line from Eq.(14). In all three curves, we use $S_\pi = 2.07$ GeV^2 corresponding to the mass squared of the Roper resonance. To check the sensitivity on S_π, we have increased the continuum threshold by 0.5 GeV^2 and plotted in the same figure denoted by respective thin lines.

In extracting some physical values, one has to fit the curves within the appropriate Borel window using the function $c + bM^2$. The unknown single pole term, b, is represented by the slope of each Borel curve. The intersection of the best fitting curve with the vertical axis gives the value of c. Figure 2 (b) shows the best fitting curves within the Borel window, $0.8 \leq M^2 \leq 1.2$ GeV^2. This window is chosen following the criteria mentioned above. But as the Borel curves are almost linear around $M^2 \sim 1$ GeV^2, the qualitative aspect of our results does not change significantly even if we use the slightly different window.

The $\gamma_5\sigma_{\mu\nu}q^\mu p^\nu$ sum rule yields $c \sim 0.00308$ GeV^6. To determine $g_{\pi N}$, the unknown parameter λ_N needs to be eliminated by combining with the nucleon odd sum rule [6]. According to the analysis in Ref. [6], this sum rule yields $g_{\pi N} \sim 10$ relatively close to its

empirical value. As can be seen from the thin solid curve which is almost indistinguishable from the thick solid curve in Fig. 2 (b), this result is not sensitive to the continuum threshold, S_π. Also note from Table I that the unknown single pole term represented by b is relatively small in this sum rule.

The result from the $i\gamma_5$ sum rule is $c \sim -0.0003$ GeV^6, which is obtained from linearly fitting the thick dot-dashed curve in Fig. 2 (a). Even though the thin dot-dashed curve is almost indistinguishable from the thick dot-dashed curve, the best fitting value for c with $S_\pi = 2.57$ GeV^2 is about 50 % smaller than the one with $S_\pi = 2.07$ GeV^2. This is because the total OPE strength of this sum rule is very small. The negative value of c indicates that $g_{\pi N}$ is negative. Also the magnitude of this is about a factor of ten smaller than the corresponding value from the $\gamma_5 \sigma_{\mu\nu} q^\mu p^\nu$ sum rule. When this result is combined with the nucleon odd sum rule, then the extracted πNN coupling would be a lot smaller than its empirical value and therefore it can not be acceptable as a reasonable prediction. As we discussed in Section II, the problem might due to the kinematical condition, Eq. (19). Though we have introduced this condition in order to achieve the independence from the coupling scheme employed, this condition inevitably combines two independent sum rules, Π_1 and Π_2 in Eq. (16), which reduces the OPE strength. This reduction makes the $i\gamma_5$ sum rule less reliable because of the cancellation of the main terms. Nevertheless, this study shows that one could get a totally different result depending on how the sum rule is constructed.

For the $i\gamma_5 \not{p}$ sum rule, the Borel curve around $M^2 \sim 1$ GeV^2 is almost a linear function of M^2. By linearly fitting the thick dashed curve ($S_\pi = 2.07$ GeV^2), we get $c \sim -0.00022$ GeV^6. But with using $S_\pi = 2.57$ GeV^2, we obtain $c \sim -0.0023$ GeV^6, a factor of ten larger in magnitude. Thus, there is a strong sensitivity on S_π which changes the result substantially. Again c is negative in this sum rule, indicating that $g_{\pi N}$ is negative. The sign of this result however depends on the Borel window chosen. Restricting the Borel window to smaller Borel masses, the extracted c becomes positive though small in magnitude. The slope of the Borel curve is also large, indicating that there is a large contribution from the undetermined single pole terms. The thin dashed curve (for $S_\pi = 2.57$ GeV^2) in Fig. 2 (b) is steeper than the thick dashed curve (for $S_\pi = 2.07$ GeV^2). In a sum rule, the larger continuum threshold usually suppresses the continuum contribution further. Since a more steeper curve is expected as we further suppress the continuum, this sum rule contains very large unknown single pole terms. This provides a very important issue which should be properly addressed in the construction of a sum rule. *The unknown single pole terms could be small or large depending on a specific sum rule one considers.*

From the three results, we showed that the extracted parameter, here c, could be totally different depending on how we construct a sum rule. Even the sign of the parameter is not well fixed. Certainly the $\gamma_5 \sigma_{\mu\nu} q^\mu p^\nu$ sum rule has nice features, such as small contributions from the continuum and the unknown single pole. And when it is combined with the nucleon odd sum rule, it provides $g_{\pi N}$ reasonably close to its empirical value [6]. But the other sum rules do not provide a reasonable or stable result. It is not clear if this is due to the lack of convergence in the OPE or due to the limitations in the sum rule method itself. To answer such questions, it would be useful to analyze the OPE side further. However our analysis raises an issue whether or not a sum rule based on one specific Dirac structure is reliable.

Still, regarding the sensitivity of S_π and the unknown single pole contribution, we can

provide a reasonable explanation based on effective model formalism developed in Section III. Results from the two sum rules, $i\gamma_5$ and $\gamma_5\sigma_{\mu\nu}q^\mu p^\nu$ structures, share similar properties. As can be seen from Table I, for the $i\gamma_5$ sum rule, the extracted c is -0.00033 GeV^6 when $S_\pi = 2.07$ GeV^2 is used. For $S_\pi = 2.57$ GeV^2, $c = -0.00016$ GeV^6. So the difference is 0.00017 GeV^6. This difference is close to the difference from the $\gamma_5\sigma_{\mu\nu}q^\mu p^\nu$ case. Furthermore, the magnitude of b is relatively close in the two sum rules. These common behaviors of the two sum rules are expected because, as we briefly mentioned in section III, their phenomenological structures for the continuum and the unknown single poles are almost the same except for the possible small term containing the derivative of the pion form factor. [See Eq. (17).] The similar slope and the similar contribution from S_π are actually related as can be seen from Eq. (30). In Eq. (30), the terms corresponding to the unknown single poles have the same relative sign between the positive- and negative-parity resonances as the terms corresponding to the continuum. If we assume that the sign of $\lambda_+ g_{\pi NN_+}$ is opposite to that of $\lambda_- g_{\pi NN_-}$, then there is a cancellation between the two resonances. Thus, with this sign assignment, we expect both terms, unknown single pole and the "step-like" continuum, contribute less to the sum rules. This is what Fig. 2 indicates. As Eq. (30) is independent of the coupling schemes, this explanation is valid even for the PV case. The sign assignment, within the PS coupling scheme, also explains the large slope and strong sensitivity of S_π in the $i\gamma_5 \not{p}$ sum rule. From Eq. (28) with the sign assignment, negative- and positive-parity resonances add up for the undetermined single pole and the continuum, yielding large contribution to the two.

This explanation for the $i\gamma_5 \not{p}$ sum rule can be changed for the PV coupling scheme. For the case with the undetermined single pole, as can be seen from Eq. (36), resonances with different parities cancel each other also for the $i\gamma_5 \not{p}$ case under the sign assignment introduced above. However, there is an additional single pole coming from $N \to N$ which could explain the large slope. Its contribution to A in Eq. (14) can be calculated to be $-1/2m$. In terms of magnitude, it contributes 50% of the LHS at $M^2 \sim 1$ GeV^2 with the opposite sign from the first term. Since c is negative as we showed in Table I, $g_{\pi N}$ is also negative. Since $b \sim g_{\pi N} A$, the unknown single pole term is positive which can explain the large and positive slope in this sum rule. As for the continuum, Eq. (37) shows that there are other contributions associated with $N^* \to N^*$ whose magnitudes can not be estimated. Even though we can not say that the large continuum only comes from adding up the positive- and negative-parity resonances, this is not contradictory to the sign assignment for $g_{\pi NN_+}\lambda_+$ and $g_{\pi NN_-}\lambda_-$. Note however that the negative sign of c is not firmly established in this sum rule for the $i\gamma_5 \not{p}$ structure as there is a possibility that c can be positive for different Borel window chosen. In this case, the positive and large slope of the Borel curve can not be well explained within the effective model.

Nevertheless, our study in this work, though it was specific to the two-point nucleon correlation function with pion, raises important issues in applying QCD sum rules in calculating various physical quantities. Most QCD sum rule calculations are performed based on a specific Dirac structure without justifying the use of the structure. As we presented in this work, a sum rule result could have a strong dependence on the specific Dirac structure one considers. This dependence is driven by the way how the sum rule is constructed or by the difference in the continuum contributions or the unknown single pole terms. The continuum and the unknown single pole terms are large in some case while they are small

in other cases.

V. SUMMARY

In this work, we have presented three different sum rules for the two-point correlation function with pion, $i \int d^4 x e^{iq \cdot x} \langle 0|T J_N(x) \bar{J}_N(0)|\pi(p)\rangle$, beyond the soft-pion limit. The PS and PV coupling scheme independence has been imposed in the construction of the sum rules. We have corrected an error in the previous sum rule in Ref. [4] and found that the sum rule contains large contribution from the unknown single pole, b, and the continuum. On the other hand, the sum rules for $i\gamma_5$ and $\gamma_5 \sigma_{\mu\nu} q^\mu p^\nu$ structures share similar properties, relatively similar contributions from the continuum and the unknown single pole. By making specific models for higher resonances, we have explained how the latter two sum rules are different from the $i\gamma_5 \not{p}$ sum rule. Within the PS coupling scheme, the difference can be well explained by the cancellation or addition of the positive- and negative-parity resonances in higher mass states. Within the PV coupling scheme, the large slope of the Borel curve in the $i\gamma_5 \not{p}$ sum rule can be attributed to the single pole coming from $N \rightarrow N$ transition even though this explanation is limited to the case with negative value of $g_{\pi N}$. The value of c extracted from the $i\gamma_5$ and $\gamma_5 \sigma_{\mu\nu} q^\mu p^\nu$ sum rules are different. For the $i\gamma_5$ sum rule, in order to eliminate the coupling scheme dependence, we need to impose the on-mass-shell condition before the matching the OPE and phenomenological correlators. Then a significant cancellation occurs and it makes the $i\gamma_5$ sum rule less reliable. We have stressed that in the construction of a sum rule, a care must be taken.

REFERENCES

[1] M.A. Shifman, A.I. Vainshtein, and V.I. Zakharov, Nucl. Phys. **B 147** (1979) 385,448.

[2] For general review see L.J. Reinders, H. Rubinstein and S. Yazaki, Phys. Rep. **127** (1985) 1.

[3] H. Shiomi and T. Hatsuda, Nucl. Phys. **A 594** (1995) 294.

[4] M. C. Birse and B. Krippa, Phys. Lett. B **373** (1996) 9; Phys. Rev. C **54** (1996) 3240.

[5] V. M. Belyaev and Ya. I. Kogan, JETP Lett. **37**, (1983) 730; B. L. Ioffe and A. G. Oganesian, Phys. Rev. D **57** (1998) R6590.

[6] Hungchong Kim, Su Houng Lee and Makoto Oka, *Los Alamos Preprint nucl-th/9809004, submitted to Physics Letters B.*

[7] X. Jin and J. Tang, Phys. Rev. D **56** (1997) 515.

[8] B. L. Ioffe, Nucl. Phys. **B188** (1981) 317.

[9] V. M. Belyaev, V. M. Braun, A. Khodjamirian and R. Rückl, Phys. Rev. D **51** (1995) 6177.

[10] B. L. Ioffe and A. V. Smilga, Nucl. Phys. **B 232** (1984) 109.

[11] V. A. Novikov, M. A. Shifman, A. I. Vainshtein, M. B. Voloshin and V. I. Zakharov, Nucl. Phys. **B 237** (1984) 525.

[12] A. A. Ovchinnikov and A. A. Pivovarov, Yad. Fiz. **48** (1988) 1135.

TABLES

TABLE I. The best-fit values for the parameters c and b obtained within the Borel window $0.8 \leq M^2 \leq 1.2$ GeV^2. The numbers in parenthesis are obtained when $S_\pi = 2.57$ GeV^2 is used.

	c (GeV^6)	b (GeV^4)
$i\gamma_5 \not{p}$	-0.00022 (-0.0023)	0.011 (0.0145)
$i\gamma_5$	-0.00033 (-0.00016)	-0.00183 (-0.0021)
$\gamma_5\sigma_{\mu\nu}q^\mu p^\nu$	0.00308 (0.002906)	0.00257 (0.0029)

FIGURES

FIG. 1. The result of Birse and Krippa's sum rule for $g_{\pi N}$ is shown in Figure (a). The solid line is obtained by eliminating the unknown single pole term using the differential operator $1 - M^2 \partial/\partial M^2$ and the dashed line is obtained simply by neglecting the unknown single pole term. Figure (b) is similarly obtained but after correcting the factors in the treatment of the continuum.

FIG. 2. (a) Borel curves obtained from the three different sum rules. The thick solid line (thick dashed line) is for the $\gamma_5 \sigma^{\mu\nu} q_\mu p_\nu$ ($i\gamma_5 \not{p}$) structure. The thick dot-dashed line is for the $i\gamma_5$ structure. Corresponding thin lines are obtained when $S_\pi = 2.57$ GeV^2 is used. (b) The curves obtained by linearly fitting the Borel curves within the range, $0.8 \le M^2 \le 1.2$ GeV^2. The thin solid lines and thin dot-dashed lines are almost indistinguishable from the corresponding thick lines.

Figure 1
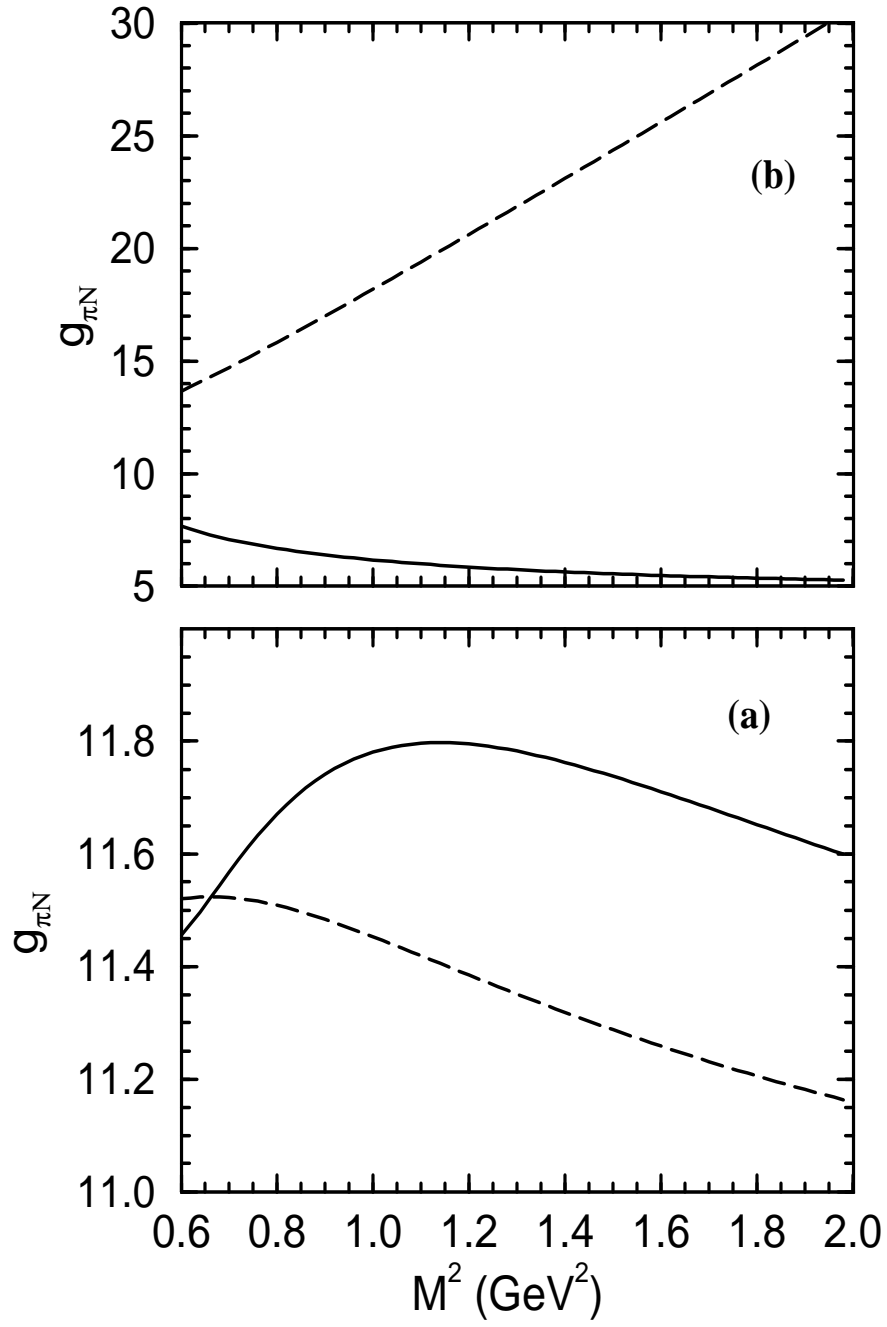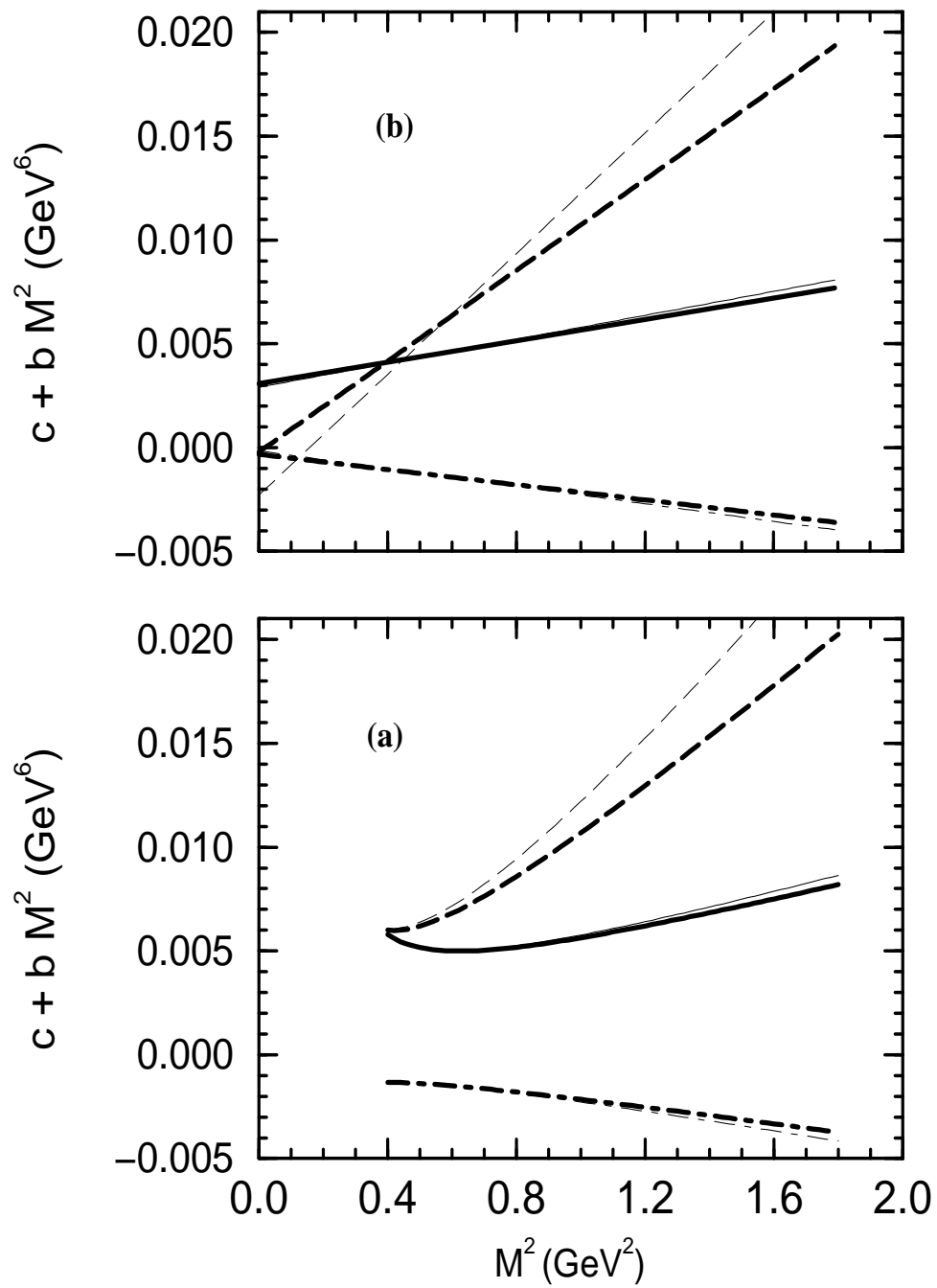
Figure 2